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Formalization of the non-linear programming for the configuration of the heat exchange system in column apparatuses of the petrochemical industry with minimum costs

Zastosowanie programowania nieliniowego dla konfiguracji systemu wymiany ciepła w aparatach kolumnowych przemysłu petrochemicznego przy minimalnych kosztach

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ABSTRACT: For increasing the efficiency of heat transfer processes, the layer-by-layer concept of arrangement of various types of packings in column apparatuses is considered. The results of hydraulic tests of a dry and irrigated cell packing, as well as individual hydraulic tests of a mesh packing, are presented. Methods and algorithms for solving linear programming problems under "uncertainty" conditions were considered. However, in some areas of science it is often difficult or impossible to formalize the problem in an appropriate way and reduce it to a linear programming problem. In this paper, methods for solving non-linear programming problems with a vector objective function are considered. At present, the use of non-linear programming in the vast majority of real situations is reduced to linear approximation models. Along with this, at a significant non-linearity, due to its specificity or influence on the nature of the model, it is necessary to apply optimization methods that are much more complex than, for example, the simplex method. However, the importance of non-linear programming is constantly increasing. This is due to the rapidly growing knowledge of managers and specialists in the use of mathematical models designed to prepare solutions, as well as the increasing availability of computer programs for solving large-scale nonlinear problems. The analysis and studies of the hydrodynamics of a number of regular packings have shown that cell and mesh packings are promising for the implementation of the phase inversion mode. The proposed concept of intensifying heat transfer processes in column apparatuses is based on the use of layers of various packings arranged in the following order in the apparatus, type 1 – cell, type 2 – mesh. At the same time, their main geometric characteristics differ significantly from each other. However, the structure of the bulk packing is inhomogeneous, which makes it difficult to implement a stable mode of local phase inversion under these conditions. This is due to the structure of column apparatuses with bulk packing, in which there is an increased proportion of pores (porosity) near the walls of the apparatus. Porosity is very significant; it can reach up to 40%, and as a result, the local velocity near the walls exceeds the velocity in the centre of the apparatus by up to 70%. By contrast, the use of regular packing structures makes this potentially highly efficient mode technically possible.

Key words: cell packing, mesh packing, column apparatus, non-linear programming, heat transfer.

STRESZCZENIE: W celu zwiększenia efektywności procesów wymiany ciepła rozważa się koncepcję warstwowego rozmieszczenia różnego rodzaju pakietów w aparatach kolumnowych, warstwa po warstwie. Przedstawiono wyniki badań hydraulicznych suchego i nawodnionego upakowania komórkowego oraz indywidualne badania hydrauliczne upakowania sieciowego. Uwzględniono metody i algorytmy dla rozwiązywania problemów programowania liniowego w warunkach „niepewności". Jednakże, w niektórych dziedzinach nauki często trudne lub niemożliwe jest sformalizowanie problemu w odpowiedni sposób i zredukowanie go do problemu programowania liniowego. W niniejszym artykule uwzględniono metody rozwiązywania problemów programowania nieliniowego z wektorową funkcją celu. Zastosowanie programowania nieliniowego w znacznej większości rzeczywistych sytuacji jest obecnie zredukowane do modeli aproksymacji liniowej. Wraz z tym, przy znaczącej nieliniowości, ze względu na jej specyfikę lub wpływ na charakter modelu, konieczne jest zastosowanie metod optymizacji, które są o wiele bardziej złożone niż na przykład metoda sympleksów. Jednakże, znaczenie programowania nieliniowego stale rośnie. Wynika to z szybko rosnącej wiedzy kadry zarządzającej i specjalistów od stosowania modeli matematycznych zaprojektowanych do przygotowywania rozwiązań, jak również zwiększającą się dostępnością programów komputerowych do rozwiązywania wielkoskalowych problemów nieliniowych. Analiza i badania hydrodynamiki pewnej liczby upakowań regularnych pokazały, że upakowania komórkowe i sieciowe są obiecujące dla realizacji trybu odwrócenia fazy. Zaproponowana koncepcja zintensyfikowania procesów wymiany ciepła w aparatach kolumnowych oparta jest o zastosowanie warstw różnych upakowań ułożonych w następującej kolejności w aparacie: typ 1 – komórka, typ 2 – sieć. Jednocześnie ich główne cechy geometryczne znacznie

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różnią się od siebie. Jednakże, struktura upakowania luźnego jest niejednorodna, co utrudnia realizację stabilnego trybu lokalnego odwrócenia fazy w tych warunkach. Jest to spowodowane strukturą aparatów kolumnowych z upakowaniem luźnym, w których blisko ścian aparatu istnieje zwiększony udział porów (porowatość). Porowatość jest bardzo znacząca; może sięgać do 40%, a w efekcie lokalna prędkość blisko ścian przekracza prędkość w środku aparatu o wartość do 70%. Dla odmiany, zastosowanie regularnych struktur upakowania czyni ten potencjalnie wysoce wydajny tryb technicznie możliwym.

Słowa kluczowe: upakowanie komórkowe, upakowanie sieciowe, aparat kolumnowy, programowanie nieliniowe, przekazywanie ciepła.

Introduction

This paper proposes a method for formalizing a non-linear programming model for the computer-aided design of a heat exchange system with minimal investment. This method is applicable to thermal processes in the chemical and petrochemical industries.

There are constructive and technological methods for intensifying heat transfer processes in column apparatuses. Constructive methods are associated with the development of more efficient packings they are discussed in the publications of Sokol et al. (2009) and Kagan et al. (2013).

For the intensification of heat transfer processes, technological methods are also of practical interest, such as the implementation of a pulsating gas flow in a column apparatus with a packing. The study of these devices is considered in the work of Kagan et al. (2013).

The pulsation of the liquid phase flow rate on time and its influence on the efficiency of heat transfer processes were studied by Memedlyaev (1994) and Moskalik (1994).

An analysis of the research results shows that the operation of column apparatuses with packing in the phase inversion mode makes it possible to provide the maximum possible intensification of the process (Memedlyaev, 1994; Moskalik, 1994).

We accept the following signs. For each hot stream i from the set of hot streams $H(i \in H)$ and each cold stream j from the set of cold streams $C(i \in C)$, the flow rates G_i and G_j, respectively, and the inlet T_i^{in} and T_j^{in} outlet temperatures are given.

For each utilization of a hot stream i from the set $H(i \in HU)$ and each utilization of a cold stream j from the set $C(i \in CU)$, the solution of the linear programming model gives information about the available thermal energy. The individual utilization of flows is determined from the solution of the mixed-integer linear programming model for each pair. These recycling flows are denoted by indices HU' and CU' respectively. In this case, the temperature change can be recorded for both the hot stream $\Delta H_i(i \in HU')$ and the cold one $\Delta H_i(j \in CU')$.

Experimental part

Since, when the column is operating in the phase inversion mode, the productivity of such a column is determined by the porosity of the packing $|e|$ (Timonin et al., 2014). It is obvious that at the corresponding flow rates of the interacting gas and liquid flows, a local regime of phase inversion is possible under counterflow conditions between the layers of packing of type 1 and type 2, where the porosity is $e_2 < e_1$. Under these conditions, it can be assumed that, in general, along the height of the column apparatus, the flow of interacting flows will be of a pulsating nature, which will also have a positive influence on the efficiency of heat transfer processes (Memedlyaev, 1994; Timonin et al., 2014).

Hydrodynamic tests were carried out, where a new design of a cell packing having a highly developed specific surface and made of metal foil was used as a layer of a regular packing type 1. The design of the proposed cell packing is based on an equilateral hexagon with 6 mm sides.

The packing is made of aluminum foil of 0.1 mm thickness. The tested packing was located in a column 196 × 196 mm made of Plexiglas. The total height of the packing in the column apparatus was 520 mm. Packing layers in the form of 4 separate blocks 85 mm high were placed in the apparatus with a gap of 10 mm relative to each other. The top layer of the cell packing had a height of 45 mm. To provide optimal conditions for breaking the liquid film between layers of packing adjacent in height, the latter are made with an offset.

The main geometric characteristics of the tested cell packing are presented in Table 1.

During experiments to study the hydraulic resistance of the cell packing, atmospheric air at a temperature of 20–22°C was used as the working gas.

Table 1. Characteristics of cell packing

Tabela 1. Charakterystyka upakowania komórkowego

Names and geometric dimensions of elements	**Specific area, *a***	**Porosity, $\|\varepsilon\|$**	**Equivalent diameter of the channel, $db = 4\varepsilon/a$**
[mm]	[m^2/m^3]	[m^2/m^3]	[m]
6 × 10 hexagon	387	0.96	0.0099

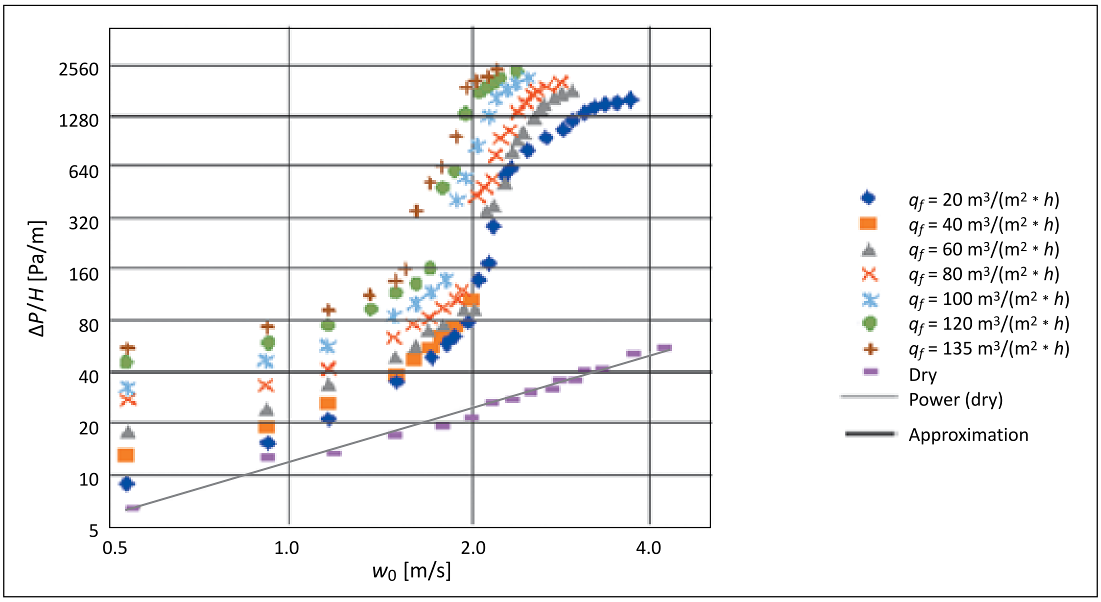

Figure 1. Dependence $f(w_0) = \Delta P / H$ for cell packing: q_f – hydraulic resistance of cell shaped irrigated filling; w_0 – fictitious velocity of the gas

Rysunek 1. Zależność $f(w_0) = \Delta P / H$ dla upakowania komórkowego: q_f – opór hydrauliczny nawodnionego wypełnienia w kształcie komórek; w_0 – prędkość pozorna gazu

Air flow was changed from 60 m^3/h to 484 m^3/h (Figure 1).

As described earlier, the practical use of an effective phase inversion mode in industrial conditions constrains a very narrow range of operation of all known bulk packings under gas loads. However, the analysis of work (Vaganov, 2011), where a mesh packing 2 was tested, shows a wider and more predictable character under gas loads in the suspension mode, which makes it possible to ensure a stable technological mode of operation of packed column apparatus, unlike any structures of bulk contact devices.

The main geometric characteristics of the tested mesh packing are presented in Table 2. The results of our experiments on measuring the hydraulic resistance of a dry and irrigated mesh packing are shown in the graphs (Figure 2).

Table 2. Characteristics of mesh packing

Tabela 2. Charakterystyka upakowania sieciowego

Names and geometric dimensions of elements	Specific area, a	Porosity, $\|\varepsilon\|$	Equivalent diameter of the channel, $db = 4\varepsilon / a$
[mm]	[m^2/m^3]	[m^2/m^3]	[m]
mesh	400	0.97	0.001

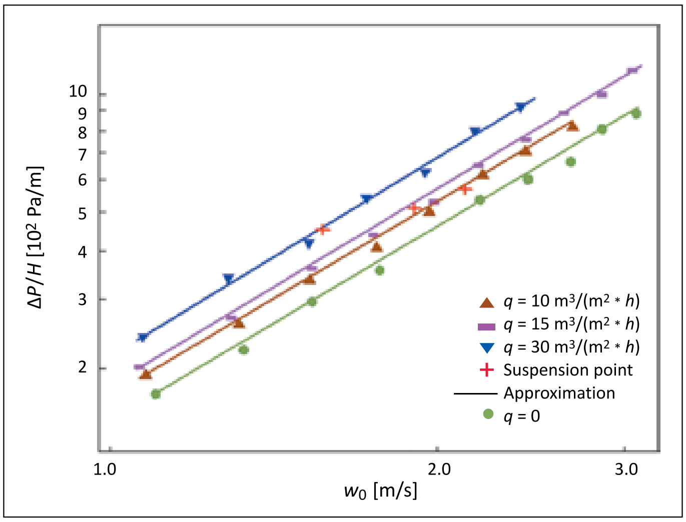

Figure 2. Graph of dependences of the hydraulic resistance of one linear metre of a regular mesh packing on the air velocity based on the total cross section of the apparatus at different values of irrigation density

Rysunek 2. Wykres zależności oporu hydraulicznego jednego metra liniowego regularnego upakowania sieciowego od prędkości powietrza w oparciu o całkowity przekrój aparatu przy różnych wartościach gęstości nawodnienia

The solution of the mixed-integer linear programming model provides information about the pairs that take place in the recycling process. Let's denote the network of pairs

$$MA = \{(i,j)\},\ i \in HT, j \in CT \tag{1}$$

where: $HT = (H) \cup (HU')$ and $CT = (C) \cup (CU')$.

From the mixed-integer linear programming model, it is possible to get information about the amount of heat for each pair (Papoulias and Grossman, 1983; Aslanov et al., 2022).

For the mathematical formulation of the non-linear programming model, the following signs are used to characterize the topology of a superstructure of substructures. Superstructure of all flows:

$$HCT = (HT) \cup (CT) \tag{2}$$

The superstructure for each flow includes a network of flows l, which is denoted by the network index $N_k = \{l\}$. Each flow from l is combined, if possible, according to the heat capacity of the flow rate f_l^k and temperature t_l^k. Also, the superstructure for each flow K includes a split $S_k \in \{s\}$ and an offset $M_k \in \{m\}$ (Abbasov et al., 2006; Habibov et al., 2022). Outline of separation and displacement flows with initial inputs and outputs of the K superstructure:

$$\left.\begin{aligned} S_k^{in}(S) &= \{l(l \in N_k)\ in\ separation\ inlet\ S\} \\ S_k^{out}(S) &= \{l(l \in N_k)\ in\ separation\ outlet\ S\} \end{aligned}\right] S \in S_k$$
$$\left.\begin{aligned} M_k^{in}(S) &= \{m(m \in N_k)\ in\ displacement\ inlet\ S\} \\ M_k^{out}(S) &= \{m(m \in N_k)\ in\ displacement\ outlet\ S\} \end{aligned}\right] m \in M_k \tag{3}$$

Input and output streams for each proposed apparatus for pairs MA:

$$\left.\begin{aligned} &E_{ij}^{H_{in}} = \{n(n \in N_i)\, inlet\ of\ hot\ stream\ i\ at\ po\mathrm{int}(i,j) \in MA\} \\ &E_{ij}^{H_{out}} = \\ &= \{p(p \in N_i)\, outlet\ of\ hot\ stream\ i\ at\ po\mathrm{int}(i,j) \in MA\} \\ &E_{ij}^{C_{in}}(m) = \\ &= \{q(q \in N_j)\, inlet\ of\ cold\ stream\ j\ at\ po\mathrm{int}(i,j) \in MA\} \\ &E_{ij}^{C_{out}}(m) = \\ &= \{r(r \in N_j)\, outlet\ of\ cold\ stream\ j\ at\ po\mathrm{int}(i,j) \in MA\} \end{aligned}\right] \tag{4}$$

Limitations can be determined from the following.

Material balance for separation

$$\begin{gathered} \Sigma f_l^k - \Sigma f_l^k = 0,\ s \in S_k,\ k \in HCT \\ l \in S_k^{in}(s),\ l \in S_k^{out}(s) \end{gathered} \tag{5}$$

Material balance for displacement:

$$\begin{gathered} \Sigma f_l^k - \Sigma f_1^k = 0,\ m \in M_k,\ k \in HCT \\ l \in M_k^{in}(m),\ l \in M_k^{out}(m) \end{gathered} \tag{6}$$

Thermal balance for displacement:

$$\begin{gathered} \Sigma f_l^k t_l^k - \Sigma f_l^k t_l^k = 0,\ m \in M_k,\ k \in HCT \\ l \in M_k^{in}(m),\ l \in M_k^{out}(m) \end{gathered} \tag{7}$$

Heat balances for the heat exchange part:

$$\left.\begin{aligned} &Q_{ij} - f_l^i\left(t_n^i - t_p^i\right) = 0,\ n \in E_{ij}^{H_{in}} \\ &p \in E_{ij}^{H_{out}},\ i \in HU' \\ &Q_{ij} - f_l^i \Delta H_i^H = 0,\ n \in E_{ij}^{H_{in}} \\ &p \in E_{ij}^{H_{out}},\ i \in HU',\ (i,j) \in MA \\ &Q_{ij} - f_l^j\left(t_q^j - t_r^j\right) = 0,\ q \in E_{ij}^{C_{out}} \\ &r \in E_{ij}^{C_{in}},\ j \in CU' \\ &Q_{ij} - f_l^j \Delta H_i^C = 0,\ q \in E_{ij}^{C_{out}} \\ &r \in E_{ij}^{C_{in}},\ j \in CU' \end{aligned}\right] \tag{8}$$

Inlet heat content:

$$f_l^k = F_k,\ l \in S_k^{in}(S),\ k \in (H) \cup (C) \tag{9}$$

where:

$$F_k = \{F_i,\ i \in H,\ F_j, j \in C\}$$

Minimum temperature limits:

$$\left.\begin{aligned} t_n^i - t_q^i \ge \Delta T_{\min},\ n \in E_{ij}^{H_{in}},\ q \in E_{ij}^{C_{out}} \\ t_p^i - t_r^i \ge \Delta T_{\min},\ p \in E_{ij}^{H_{in}},\ r \in E_{ij}^{C_{out}} \end{aligned}\right] (i,j) \in MA \tag{10}$$

Inlet and outlet temperatures:

$$\left.\begin{aligned} t_l^k &= T_k^{in}\ \ l \in S_k^{in}(s) \\ t_l^k &= T_k^{out}\ \ l \in M_k^{out}(m),\ k \in HCT \end{aligned}\right] \tag{11}$$

where:

$$\left.\begin{aligned} T_k^{in} &= \{T_i^{in}, i \in H, T_j^{in}, j \in C, T_j^{H_{in}}, i \in HU', T_j^{C_{in}}, j \in CU'\} \\ T_k^{out} &= \{T_i^{out}, i \in H, T_j^{out}, j \in C, T_j^{H_{out}}, i \in HU', T_j^{C_{out}}, j \in CU'\} \end{aligned}\right]$$

Separation inlet and outlet temperature equation:

$$\begin{gathered} t_l^k = t_p^k,\ l \in S_k^{in}(s) \\ p \in S_k^{out}(s);\ s \in S_k,\ k \in HCT \end{gathered} \tag{12}$$

Non-negative constraints:

$$f_1^1 \ge 0.1 \in N_k,\ k \in HCT \tag{13}$$

Finally, the heat exchange area can be reduced with respect to a given heat flow Q and flow temperatures, i.e. (Chizh et al., 2014):

$$A_{ij} = Q_{ij} U_{ij}^{-1} (LMTD)_{ij}^{-1} \tag{14}$$

Where U_{ij} is the heat transfer coefficient for the pair $(f) \in MA$, $(LMTD)i_j^{-1}$ the average logarithmic temperature difference for the pair (i, j).

Thus, the objective function for minimizing capital investment can be written as:

$$\min \Sigma C_{ij} A_{ij}\left(i,j\right) \in MA \tag{15}$$

where: C_{ij} the cost A_{ij} factor comes from (14) (Wood et al., 1985).

Conclusions

The proposed method can significantly reduce the heat exchange surface area and, as a result, obtain an objective function to minimize capital investments. For this purpose, the following tasks were solved:

1. a mathematical model consisting of all the main technological, including hydrodynamic, thermal and design parameters of the apparatus has been developed;
2. a decomposition approach for solving the design problem in the form of a two-level optimization procedure is proposed.

The developed algorithm allows having various alternative configurations for each specific case. The final structure of the packing system is formed by the decomposition method from substructures for each predicted pair of packings. At the lower level, the device parameters providing the extremum of the technical and economic criterion and satisfying the constraints are selected. At the highest level, a selection of pairs of packings for synthesizing the mass transfer structure is carried out.

The proposed approach to solving the problem can be successfully applied in the development of packing systems for technological processes in the chemical, petrochemical, oil refining, power, metallurgical and food industries.

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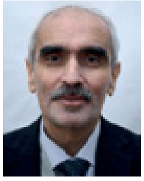

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