

# Rheological properties of elastic-visco-plastic liquid

## Właściwości reologiczne cieczy sprężysto-lepkoplastycznej

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**ABSTRACT:** A model for the motion of a compressible elastic-viscous-plastic fluid in a round pipe is proposed. The main indicators of the flow, volume flow and speed are obtained. Numerous hydrodynamic processes are associated with the properties of liquids. It is known that the mechanical and physical properties of liquids can be described by various models. Various models have been proposed that partially describe the processes of hydromechanics. The proposed model makes it possible to qualitatively describe the deformation processes that occur in various systems. To describe a closed theory of motion of a continuous medium and, in particular, between stress and strain, it is necessary to have a mechanical model. From the literature analysis it is known that there are simple models, which include elastic, viscous and plastic. At the same time, it should be noted that the mechanical model partially shows the mechanical state of the elastic body in the form of Hooke's law. Practice shows that there are more complex liquids that do not obey this law. One of these fluids is an elastic-viscous-plastic fluid. In this work, a model is proposed that describes the elastic-viscous-plastic properties of liquids, which sequentially connects the elastic and viscous-plastic elements. For such a medium, the total resistance will be the sum of the stress corresponding to the elastic deformation and the stress caused by the viscous-plastic resistance. Based on the proposed model, an equation is obtained taking into account the coefficient of volumetric elastic expansion, and an equation is obtained for determining the velocity distribution over the pipe section and volume flow for a given liquid. Calculations have shown that with an increase in compressibility, the flow rate of the liquid partially increases, which in some practical cases produces a positive effect.

**Key words:** elasticity, viscoelasticity, elastic expansion, deformation, surfactants.

**STRESZCZENIE:** W artykule zaprezentowano model ruchu ściśliwej cieczy sprężysto-lepkoplastycznej w rurze o przekroju okrągłym. Uzyskano główne wskaźniki przepływu, przepływu objętościowego i szybkości. Liczne procesy hydrodynamiczne są związane z właściwościami cieczy. Wiadomo, że właściwości mechaniczne i fizyczne cieczy można opisać za pomocą różnych modeli. Zaproponowano modele, które częściowo opisują procesy mechaniki cieczy. Przedstawiony model umożliwia jakościowy opis procesów odkształcenia zachodzących w różnych systemach. Model mechaniczny jest konieczny do opisania zamkniętej teorii ruchu ośrodka ciągłego, w szczególności pomiędzy naprężeniem i odkształceniem. Z analizy literatury wiadomo, że istnieją proste modele obejmujące ciecze sprężyste, lepkie i plastyczne. Należy jednocześnie zauważyć, że model mechaniczny częściowo przedstawia stan mechaniczny ciała sprężystego, wykorzystując prawo Hooke'a. Praktyka pokazuje, że istnieją ciecze bardziej złożone, niezachowujące się zgodnie z tym prawem. Jednym z takich płynów jest płyn sprężysto-lepkoplastyczny. W niniejszej pracy zaproponowano model opisujący właściwości sprężysto-lepkoplastycznej cieczy, który sekwencyjnie łączy elementy sprężyste i lepkoplastyczne. W przypadku takiego ośrodka opór całkowity jest sumą naprężenia odpowiadającego odkształceniu sprężystemu i naprężenia spowodowanego przez opór lepkoplastyczny. Na podstawie zaproponowanego modelu uzyskano równanie uwzględniające współczynnik objętościowej rozszerzalności sprężystej, a także uzyskano równanie do określenia rozkładu szybkości na przekroju rury oraz przepływu objętościowego dla danej cieczy. Obliczenia pokazały, że ze wzrostem ściśliwości szybkość przepływu cieczy częściowo wzrasta, co w pewnych praktycznych przypadkach daje pozytywny efekt.

**Słowa kluczowe:** sprężystość, lepkosprężystość, rozszerzalność sprężysta, odkształcenie, środki powierzchniowo czynne.

### Introduction

It is known that the mechanical properties of various liquids can be described by various models. Such models make it possible to qualitatively describe the process of deformation that occurs in various systems. Note that in order to build a closed

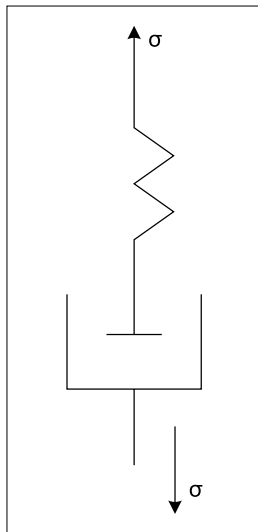
theory of the motion of a medium, the relationship between the kinematic and dynamic states of a particle and, in particular, between stresses and strains, expressed using a mechanical model, must be known.

Note that simple models include elastic, viscous and plastic (Mirzajanzade and Shirinzade, 1986; Dmitriev and Kadet,

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2016). In this case, it is necessary that the mechanical equations of the state of an elastic body are expressed using Hooke's law. However, more complex models also exist. One of these fluids is an elastic-viscous-plastic fluid. In early works, the simplest mechanical models, illustrating the mechanical equations of the state of a viscous, elastic and plastic body were presented. By combining these data, as well as the simplest models, various complex environments can be described. Thus, an elastic-viscous-plastic medium can be characterized by a model in which elastic and viscous-plastic elements are connected in a series (Figure 1).



**Figure 1.** Mechanical model of an elastic-viscous-plastic fluid  
**Rysunek 1.** Model mechaniczny płynu sprężysto-lepkoplastycznego

For such a medium, the total stress will be the sum of the stress corresponding to elastic deformation and the stress caused by viscous-plastic resistance.

### Rheological model

With an increase in the depth of oil and gas condensate wells, bottom-hole temperatures and pressures increase significantly, which actively affects the physicochemical properties of drilling fluids, grouting cements and non-Newtonian fluids (Mirzajanzade and Shirinzade, 1986).

It should be noted that the use of various fluids in the country, as well as abroad, shows that many problems of drilling and well operation can be successfully solved using artificial fluids. These fluids have been used to control the parameters of flushing and pumped fluids. Synthetic fluids are used to stabilize and control the rheological properties of such fluids. At the same time, stabilization and increase in the thermal stability of such liquids is achieved by adding polymeric surfactants.

Practice shows that there is a certain hydrodynamic interaction between the formation, production and drilling wells. So, in particular, the continuous filtration of the drilling fluid into the reservoir through the mud cake formed on the surface of the formation leads to a change in pressure in the bottomhole zone of the well. This changing pressure is taken as the value of the dynamic reservoir pressure, which in some cases differs significantly from the static pressure.

Dynamic reservoir pressure and the duration of its equalization in the bottomhole zone and reservoir has a more significant impact on the occurrence of various accidents and major complications in drilling. The faster the pressure in the reservoir and the bottomhole zone equalizes, the less the risk of complications in the well.

The duration of the equalization of dynamic reservoir pressure in the reservoir and bottomhole zone of the well depends on many parameters, including the rheological properties of elastic-viscous-plastic fluids.

Field practice shows that the pressure equalization time largely depends on the lithological composition of the rocks. It should be noted that when opening permeable clayey rocks, or the presence of a clayey fraction in rocks, a near-wall layer of high viscosity and elasticity is formed near the walls of the filtration channels. Since the mud used in drilling has viscous-plastic properties, such a liquid also has compressible properties.

However, when considering the elastic-viscous-plastic properties of the clay cake and rocks, this picture changes dramatically. For example, if the mud cake and rock have viscous and elastic properties, then this can lead to a significant increase in the dynamic pressure equalization time. In this case, this does not happen instantly with a change in pressure, but with some delay, that is, the deformation of the liquid has a relaxation character. This process is confirmed by numerous experiments.

In this regard, it becomes necessary to take into account the elastic-viscous properties of the drilling fluid in hydrodynamic calculations. Note that the viscoelastic properties of oils can be used when drilling in formations with anomalous oils.

Numerous studies have shown that some liquids are elastic-viscous-plastic media. Such liquids do not obey Newton's law (Vulis and Kashkarov, 1965).

The deformation behaviour of fluids with anomalous, elastic-viscous-plastic properties is most fully characterized by three parameters: compressibility, structural viscosity, and ultimate shear stress.

To solve this problem, we consider the rectilinear stationary motion of an elastic-viscous-plastic fluid in a round cylindrical pipe. Note that in this case, when solving problems, as well as determining the hydraulic resistance, the influence of the initial section is not taken into account (Lovkis, 2012).

According to the linear law of motion of a compressible fluid in one-dimensional motion, the volume elasticity of fluid expansion (Shchelkachev and Lapuk, 2001; Kelbaliev et al., 2017).

$$\beta = -\frac{dV}{Vdp} \quad (1)$$

However

$$V = \frac{m}{\rho} \quad \text{or} \quad dV = -\frac{md\rho}{\rho^2} \quad (2)$$

Solving jointly, we have:

$$\beta = \frac{d\rho}{\rho dp} \quad (3)$$

Let's define the pressure gradient:

$$\frac{dp}{dx} = \frac{1}{\beta\rho} \frac{d\rho}{dx} \quad (4)$$

It is known that the Shvedov-Bingham equation can be written as

$$\tau = -\mu \frac{dv}{dr} + \tau_0 \quad (5)$$

Given the above, we write the balance of forces acting on the cylindrical fluid element:

$$\pi r^2 \Delta p = 2\pi r \tau dx \quad (6)$$

We solve the final equation for the pressure gradient and the Shvedov-Bingham equation (Barnes, 2003):

$$\frac{dp}{dx} = \frac{2}{r} \left( \tau_0 - \mu \frac{dv}{dr} \right) \quad (7)$$

Taking into account (4), we find:

$$\frac{1}{\beta\rho} \frac{d\rho}{dx} = \frac{2}{r} \left( \tau_0 - \mu \frac{dv}{dr} \right) \quad (8)$$

For the movement of fluid in a pipe, we solve this equation for velocity:

$$dv = -\frac{1}{\mu} \left( \frac{1}{2\beta\rho} \frac{d\rho}{dx} r - \tau_0 \right) dr \quad (9)$$

Integrating this equation from the core radius ( $r$ ) to the tube radius ( $R$ ), we obtain the following expression (Baishev et al., 2004; Yentov and Glivenko, 2008):

$$v = \frac{1}{\mu} \left[ \frac{1}{4\beta\rho} \frac{d\rho}{dx} (R^2 - r^2) - \tau_0 (R - r) \right] \quad (10)$$

Taking into account the volumetric flow rate of the core and the annular space, it is possible to determine:

$$Q = Q_{core} + Q_{annular} \quad (11)$$

Solving together, we find:

$$Q = \frac{\pi R^4}{2\mu} \cdot \left[ \frac{1}{4\beta\rho} \frac{\Delta p}{L} \left( 1 - \frac{\Delta p_0^4}{\Delta p^4} \right) + \frac{2\Delta p_0}{3L} \left( \frac{\Delta p_0^3}{\Delta p^3} - 1 \right) \right] \quad (12)$$

According to the work of Shchelkachev and Lapuk (2001), we have:

$$\Delta p = \rho_{init} - \rho_{fin} = \rho_0 \beta \Delta p (1 + \beta \bar{p}) \quad (13)$$

Thereat

$$\bar{p} = \frac{p_{init} + p_{fin}}{2}$$

Thus, the equation of an elastic-viscous-plastic fluid can be written:

$$Q = \frac{\pi R^4 \Delta p}{8\mu L} \cdot \left[ \left( 1 - \frac{\Delta p_0^4}{\Delta p^4} \right) (1 + \beta \bar{p}) + \frac{4}{3} \frac{\Delta p_0}{\Delta p} \left( \frac{\Delta p_0^3}{\Delta p^3} - 1 \right) \right] \quad (14)$$

In the case when  $\beta = 0$ , we have the Shvedov-Bingham formula, and in the case of  $\Delta p_0 = 0$ , we have the Poiseuille formula.

The proposed calculation model makes it possible to determine numerous practical tasks, that is, to determine the average speed, friction head loss, and the generalized Reynolds number (Truesdell, 1977; Brill and Mukherjee, 1999; Natanson, 2007).

To simplify, this equation can be written as:

$$Q = \frac{\pi R^4 \Delta p}{8\mu L} \left[ (1 + \beta \bar{p}) - \frac{4}{3} \frac{\Delta p_0}{\Delta p} \right] \quad (15)$$

As can be seen, with an increase in the coefficient of volumetric elastic expansion, the flow rate of the liquid increases. Depending on the nature of the compressible fluid, under certain conditions elasticity dominates over viscoelasticity (Somerton, 2014).

This model is one of the first works in the field of elastic-viscous-plastic fluids. The results obtained can also be used when considering a number of technological processes, which are currently among the current interests of the author.

## Conclusions

1. A model for an elastic-viscous-plastic fluid taking into account the coefficient of volumetric elastic expansion is proposed.
2. An equation to determine the velocity distribution over the pipe section and the volumetric flow rate for a given liquid was obtained.

3. With an increase in compressibility, the flow rate of the liquid partially increases, which in some practical cases produces a positive effect.

**Notations**

$\beta$  – coefficient of volumetric elastic expansion,  
 $m$  – is the mass of the liquid in question,  
 $\rho$  – is the density of the compressible liquid,  
 $V$  – volume of compressible liquid,

$\frac{dp}{dx}$  – pressure gradient,

$\tau$  – tangential shear stress,  
 $\tau_0$  – initial tangential shear stress,

$\frac{dv}{dr}$  – speed gradient,

$Q$  – volume flow,  
 $Q_{core}$  – fluid flow in the core of the flow,  
 $Q_{annular}$  – fluid flow in the annular space around the core,  
 $R$  – pipe radius,  
 $\Delta p$  – pressure drop,  
 $\mu$  – dynamic viscosity,  
 $\Delta p_0$  – pressure difference at which the given liquid in a pipe with a radius ( $R$ ) begins to move,  
 $L$  – pipe length,  
 $\bar{p}$  – average pressure,  
 $p_{init}$  and  $p_{fin}$  – initial and final pressure,  
 $\rho_{init}, \rho_{fin}$  – respectively, the density of the liquid at the beginning and end of the pipe,  
 $\rho_0$  – is the density of the liquid under atmospheric conditions,  
 $v$  – the speed of the fluid.

**References**

Baishev E.V., Glivenko E.V., Gubar V.A., Yentov V.M., Ershov T.B., 2004. Gas-Pulse Action on the Well Bottomhole Zone. *Fluid Dynamics*, 39: 582–588. DOI: 10.1023/B:FLUI.0000045673.89081.a9.

Barnes H.A., 2003. A review of the rheology of filled viscoelastic. *Rheology Reviews, The British Society of Rheology*, 2: 1–36.  
 Brill J.P., Mukherjee H., 1999. Multiphase flow in well. *Society of Petroleum Engineers Inc., SPE Monograph Series*, 17: 1–164. ISBN: 978-1-55563-080-5.  
 Dmitriev N.M., Kadet V.V., 2016. Hydraulics and oil and gas hydro-mechanics. *Izhevsk*: 1–352.  
 Kelbaliev G.I., Rasulov S.R., Rzaev A.G., Mustafayeva G.R., 2017. Rheology of Structured Oils. *Journal of Engineering Physics and Thermophysics*, 90(4): 996–1002. DOI: 10.1007/s10891-017-1649-z.  
 Lovkis Z.V. (ed.), 2012. Hydraulics. *Izhevsk*: 1–439.  
 Mirzajanzade A.Kh., Shirinzade S.A., 1986. Improving the Efficiency and Quality of Deep Well Drilling. *Nedra*: 1–277.  
 Natanson I.P., 2007. Short Course in Higher Mathematics. SPb-M. *Krasnodar. Ed. Lan*: 1–486.  
 Shchelkachev V.N., Lapuk B.B., 2001. Underground hydraulics. *Izhevsk*: 1–763.  
 Somerton W.H., 2012. Thermal properties and temperature-related behavior of rock/fluid system. *Developments in Petroleum Science, Elsevier*, 37: 1–266.  
 Truesdell C., 1977. A First Course in Rational Continuum Mechanics. *1st Edition. Elsevier*: 1–304. eBook ISBN 9781483220482.  
 Vulis L.A., Kashkarov V.P., 1965. Theory of viscous fluid jets. *Nauka Publishing House*: 1–432.  
 Yentov V.M., Glivenko E.V., 2008. Continuum mechanics and its application in gas and oil production. *LLC Nedra-Business Center*: 1–204.



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