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Algorithm of migration MG(F-K) in orthorhombic medium

Introduction

This article presents a description of properties of anisotropic orthorhombic medium by means of motion equations. This type of medium may be, in its relatively simple variant, a combination of transversely isotropic medium of vertical axis of symmetry, i.e. VTI and the system of parallel crevices and cracks located in vertical plain. Therefore, the orthorhombic model may be a combination of VTI model and HTI (*Horizontal Transversely Isotropy*) [7] (Fig. 1). This kind of medium is a significantly better approximation of reality than model HTI, although it must still be treated as a stage in searching for effective tools describing an azimuthal anisotropic model.

Decomposition of a complex orthorhombic model combined with definition of directions of situation and inclination of the plane of the cracks is an essential task in oil prospecting, which allows to settle several problems concerning hydrodynamics of deposit fluids and construction of deposit reservoirs. Alternatively, an orthorhombic model may function as a combination of model VTI and HTI – systems of complex internal anisotropy. In the article two cases of measurements of the wavefield will be considered along the symmetry axis; when the symmetry axis is perpendicular and when it is parallel to the lamination (cracking) plains. In both cases, appropriate motion equations will be developed and dispersion relations in order to determine vertical wavenumbers in function of horizontal wavenumber and anisotropic parameters. The mentioned vertical wavenumbers will be used in construction of algorithms MG(F-K) of migration functioning in dual domains: the wavenumbers (K) and frequency (F) and space-time – domain (*t*-*x*) [4, 5].

Basic equations

We will consider a medium as a combination of VTI model (*Vertical Transversely Isotropy*) and HTI model (*Horizontal Transversely Isotropy*). In the first case we will assume that the measurement is made along the symmetry axis x in parallel direction to lamination (crack plain) (Fig. 1).

With this assumption, the matrix of elastic modules D will be the weighted sum of matrices of both types of anisotropy, i.e. VTI and HTI. The weighted sum coefficient should be selected so as to become the VTI medium after being turned by 90° in relation to axis x medium HTI together with motionless medium VTI. Therefore, we should expect the following condition to be fulfilled:

$$\underset{\theta \to 0^o}{\frac{1}{2}\left(D^{\theta=0^o,\theta} + D^{\phi=0^o,\theta=0^o}\right)} \to C_{VTI} \qquad (1)$$

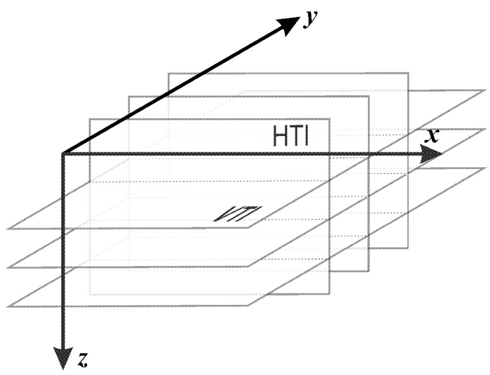

Fig. 1. Drawing of a orthorhombic model as a combination of VTI model and a medium turned to angle $\theta = 90^o$ HTI. Measurement in the plain x-z

where the matrix for VTI model is marked as C_{VTI} and this matrix fulfils relation $D^{\phi=0^o,\theta=0^o} = C_{VTI}$.

Matrix $D^{\phi=0^o,\theta=90^o}$ is a symmetrical 6 × 6 matrix and it can be shown in the following way [6]:

$$D^{\phi=0^o,\theta=90^o} = \begin{vmatrix} C_{11} & C_{13} & C_{12} & 0 & 0 & 0 \\ C_{13} & C_{33} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{13} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{vmatrix} \tag{2}$$

So the overall matrix D will be:

$$D = \frac{1}{2}\left(D^{\phi=0^o,\theta=90^o} + C_{\backslash VTI}\right) = \frac{1}{2}\begin{vmatrix} C_{11}+C_{11} & C_{12}+C_{13} & C_{12}+C_{13} & 0 & 0 & 0 \\ C_{12}+C_{13} & C_{11}+C_{33} & C_{13}+C_{13} & 0 & 0 & 0 \\ C_{12}+C_{13} & C_{13}+C_{13} & C_{11}+C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}+C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}+C_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44}+C_{66} \end{vmatrix} \tag{3}$$

We will consider motion equations for the components of the displacement field $U_{1(x)}$ and $U_{3(z)}$ disregarding the component $U_{2(y)}$ independent of components U_x and U_z and assuming on account of measurement in direction of symmetry axis x that all derivatives relative to y equal zero.

The motion equations have the form (disregarding the external force):

$$T_{ij,j} = \rho \frac{\partial^2 U_i}{\partial t^2} \tag{4}$$

where:

t – time,

ρ – medium density,

T_i – stress tensor relating to strain E_{lk} in accordance with Hooke`s law

$$T_{ij} = d_{ijlk} E_{lk} = d_{ijlk} E_{kl} \tag{5}$$

Where strain tensor $E_{lk} = E_{kl}$ is connected with displacement field components U_l by relation:

$$E_{lk} = E_{kl} = \frac{1}{2}\left(U_{l,k} + U_{k,l}\right) \tag{6}$$

Taking into account the relation (6), relation (5) can be written as matrix:

$$\begin{vmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{23} \\ T_{31} \\ T_{12} \end{vmatrix} = D \begin{vmatrix} U_{x,x} \\ 0 \\ U_{z,z} \\ 0 \\ U_{z,x} + U_{x,z} \\ 0 \end{vmatrix} \tag{7}$$

Where symmetrical matrix (6 × 6) D contains components of fourth order tensor d_{ijlk} in Voigt`s brief notation, in accordance with relation (3).

Making use of equation (7) we obtain:

$$\begin{aligned} T_{11} &= \frac{1}{2}\left\{\left(C_{11}+C_{11}\right)U_{x,x} + \left(C_{12}+C_{13}\right)U_{z,z}\right\} \\ T_{13} &= \frac{1}{2}\left(C_{44}+C_{66}\right)\left(U_{z,x}+U_{x,z}\right) \end{aligned} \tag{8}$$

and

$$T_{31} = \frac{1}{2}(C_{44} + C_{66})(U_{z,x} + U_{x,z})$$
$$T_{33} = \frac{1}{2}(C_{12} + C_{13})U_{x,x} + \frac{1}{2}(C_{11} + C_{33})U_{z,z} \tag{9}$$

From relation (4) and (8) and (9) we receive motion equations for components U_x and U_z:

$$C_{11}U_{x,xx} + \frac{1}{2}(C_{44} + C_{66})U_{x,zz} + \frac{1}{2}[C_{12} + C_{13} + C_{44} + C_{66}]U_{z,xz} = \rho\frac{\partial^2 U_x}{\partial^2 t} \tag{10}$$

$$\frac{1}{2}(C_{12} + C_{13} + C_{44} + C_{66})U_{x,zx} + \frac{1}{2}(C_{11} + C_{33})U_{z,zz} + \frac{1}{2}(C_{44} + C_{66})U_{z,xx} = \rho\frac{\partial^2 U_z}{\partial^2 t} \tag{11}$$

By presenting equations (10)-(11) in the form of Fourier transform we obtain:

$$\begin{vmatrix} C_{11}k_x^2 + \frac{1}{2}(C_{44} + C_{66})k_z^2 - \rho\omega^2 & \frac{1}{2}[C_{12} + C_{13} + C_{44} + C_{66}]k_x k_z \\ \frac{1}{2}[C_{12} + C_{13} + C_{44} + C_{66}]k_x k_z & \frac{1}{2}(C_{44} + C_{66})k_x^2 + \frac{1}{2}(C_{11} + C_{33})k_z^2 - \rho\omega^2 \end{vmatrix} \cdot \begin{vmatrix} U_x \\ U_z \end{vmatrix} = 0 \tag{12}$$

where:
k_x and k_z – wavenumbers, horizontal and vertical,
ω – angular frequency.

If we disregard shear waves qSH and qSV type assuming that $C_{66} = 0$ and $C_{44} = 0$, then from the equation determinant (12) the following form is obtained:

$$k_z^2\left[\frac{1}{2}(C_{11}^2 + C_{11}\cdot C_{33})k_x^2 - \frac{1}{2}(C_{11} + C_{33})\rho\omega^2 - \frac{1}{4}(C_{12} + C_{13})^2 k_x^2\right] - \rho\omega^2 C_{11}k_x^2 + \rho^2\omega^4 = 0 \tag{13}$$

From equation (13) we will define the square of the vertical wavenumber:

$$k_z^2 = \frac{\rho^2\omega^4 - \rho\omega^2 C_{11}k_x^2}{\frac{1}{2}(C_{11} + C_{33})\rho\omega^2 + \left[\frac{1}{4}(C_{12} + C_{13})^2 - \frac{1}{2}(C_{11}^2 + C_{11}\cdot C_{33})\right]k_x^2} \tag{14}$$

marking

$$\frac{1}{2}(C_{11} + C_{33}) = C_{33}'$$
$$\frac{1}{2}(C_{12} + C_{13}) = C_{13}' \tag{15}$$

and assuming, after Thomsen [8] that:

$$\frac{C_{11} - C_{33}'}{2C_{33}'} = \varepsilon' \tag{16}$$

and

$$\left(\frac{C_{13}'}{C_{33}'}\right)^2 = 2\delta' + 1$$
$$\frac{C_{11}}{C_{33}'} = 2\varepsilon' + 1 = q' \tag{17}$$

We receive from (14) the following relation:

$$k_z^2 = \frac{\omega^4 S_0^4 - \omega^2 S_0^2 q' k_x^2}{\omega^2 S_0^2 - \eta' k_x^2} \tag{18}$$

where:

$$\eta' = 2(\varepsilon' - \delta') \tag{19}$$

and slowness $$S_0 = \frac{1}{V_0}$$

It is easy to notice that the form of vertical wavenumber k_z is similar to wavenumber for medium VTI [1]. It only differs in the values ε and δ which were replaced with ε', δ' and q' in accordance with the definition of relations (16)-(17) and in consequence, the mean component values $C_{33}^{'}$ and $C_{13}^{'}$. The result of the first relation of equation (15) is:

$$V_0^2 = \frac{1}{2}\left(V_{II}^2 + V_{\perp}^2\right) \tag{20}$$

Therefore, another difference is in the definition of used velocities. Vertical velocity V_0 of longitudinal waves has been replaced by the root of the mean sum of velocity squares in direction of laminations and in perpendicular direction to laminations. This result seems understandable, considering the fact that for the same measurement direction in medium HTI [2] longitudinal waves propagated with velocity V_{II}, which is in the same way as in isotropic medium, while in the orthorhombic model immersed in model VTI, beside velocity in parallel direction to lamination V_{II} there is also vertical velocity $V_{\perp}$ – perpendicular to lamination.

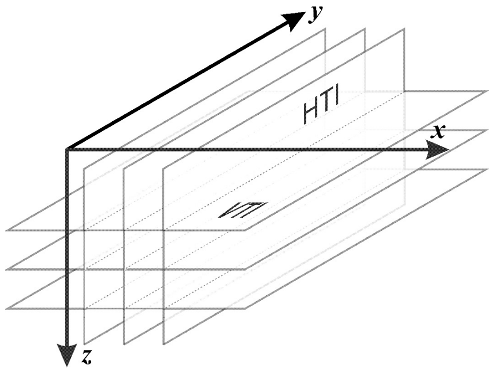

Fig. 2. Drawing of an orthorhombic medium as combination of the medium of horizontally layered VTI and layered in perpendicular direction (or crack) type HTI

Let us consider another case of orthorhombic medium, when the measurement is made along the symmetry axis of medium HTI immersed in medium VTI (Fig. 2).

The composition will be defined by matrices of elastic modules C for medium VTI and matrices of medium HTI – in such a way that the sum of these matrices for angle $\theta = 0^o$ was also equal to the matrix for the horizontally layered medium, i.e. C_{VTI}:

$$\begin{gathered} D^{\phi=90^o,\theta=0} = \frac{1}{2} C_{VTI} \\ \frac{1}{2} D^{\phi=90^o,\theta=90^o} + \frac{1}{2} C_{VTI} = S \end{gathered} \tag{21}$$

That is

$$S_{\theta \to 0} \to C_{VTI} \tag{22}$$

Using matrix $D^{\phi=90^o,\theta=90^o}$ [2] and matrix C_{VTI} for VTI of the medium we obtain:

$$S = \frac{1}{2}\begin{vmatrix} C_{11}+C_{33} & C_{12}+C_{13} & C_{13}+C_{13} & 0 & 0 & 0 \\ C_{12}+C_{13} & C_{11}+C_{11} & C_{12}+C_{13} & 0 & 0 & 0 \\ C_{13}+C_{13} & C_{12}+C_{13} & C_{11}+C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}+C_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}+C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44}+C_{66} \end{vmatrix} \tag{23}$$

In this case the measurement will be made in the plane x-z along the symmetry axis, that is in the same way as previously, we disregard calculations of component U_y and derivatives with regard to y.

By using matrix S (23), we receive the following forms for stress derivatives $T_{ij,j}$:

$$\begin{aligned} T_{11,1} &= \frac{1}{2}(C_{11}+C_{13})U_{x,xx} + C_{13}U_{z,zx} \\ T_{13,3} &= C_{44}(U_{z,xz}+U_{x,zz}) \\ T_{31,1} &= C_{44}(U_{z,xx}+U_{x,zx}) \\ T_{33,3} &= C_{13}U_{x,xz} + C_{11}U_{z,zz} \end{aligned} \tag{24}$$

By placing relation (24) in motion equations we obtain the stress forms for the components of displacement field U_x and U_z:

$$\frac{1}{2}(C_{11}+C_{13})U_{x,xx} + C_{44}U_{x,zz} + (C_{13}+C_{44})U_{z,zx} = \rho\frac{\partial^2 U_x}{\partial t^2} \tag{25}$$

$$(C_{13}+C_{44})U_{x,zx} + C_{44}U_{z,xx} + C_{11}U_{z,zz} = \rho\frac{\partial^2 U_z}{\partial t^2} \tag{26}$$

After Fourier transform has been used ($x \to k_x$, $z \to k_z$, $t \to \omega$), a matrix equation is received:

$$\begin{vmatrix} \frac{1}{2}(C_{11}+C_{33})k_x^2 + C_{44}k_z^2 - \rho\omega^2 & (C_{44}+C_{13})k_xk_z \\ (C_{44}+C_{13})k_xk_z & C_{11}k_z^2 + C_{44}k_x^2 - \rho\omega^2 \end{vmatrix} \cdot \begin{vmatrix} U_x \\ U_z \end{vmatrix} = 0 \tag{27}$$

Calculating the determinant of equation (27), we receive a dispersion relation:

$$k_z^4 a_0 + k_z^2 a_1 + a_2 = 0 \tag{28}$$

where:

$$\begin{aligned} a_0 &= C_{11}\cdot C_{44} \\ a_1 &= \frac{1}{2}(C_{11}+C_{33})C_{11}k_x^2 + C_{44}k_x^2 - (C_{44}+C_{13})^2k_x^2 - \rho\omega^2(C_{11}+C_{44}) \\ a_2 &= \frac{1}{2}C_{44}(C_{11}+C_{33})k_x^4 - C_{44}\rho\omega^2k_x^2 - \frac{1}{2}(C_{11}+C_{33})\rho\omega^2k_x^2 + \rho^2\omega^4 \end{aligned} \tag{29}$$

If we disregard the shear wave *qSV* and assume $C_{44} = 0$, then:

$$k_z^2 = -\frac{a_2}{a_1} = \frac{\rho^2\omega^4 - \frac{1}{2}(C_{11}+C_{33})\rho\omega^2k_x^2}{C_{13}^2k_x^2 + C_{11}\rho\omega^2 - \frac{1}{2}(C_{11}+C_{33})C_{11}k_x^2} \tag{30}$$

Making use of the relations:

$$\begin{aligned} & C_{11} + C_{33} = 2C_{33}(1+\varepsilon) \\ & C_{11}\cdot C_{33} = \rho^2V_\perp^4(1+2\varepsilon) = \rho^2V_\perp^4\cdot q \\ & C_{13}^2 = (1+2\delta)C_{33}^2 \\ & C_{11} = C_{33}\cdot(2\varepsilon+1) = C_{33}\cdot q \\ & \frac{1}{2}C_{11}(C_{11}+C_{33}) = \rho^2V_\perp^4(1+\varepsilon)\cdot q \end{aligned} \tag{31}$$

We obtain an expression of the square of vertical wavenumber in the form

$$k_z^2 = \frac{\omega^4S_\perp^4 - \omega^2S_\perp^2\cdot(1+\varepsilon)k_x^2}{\omega^2S_\perp^2 - (\eta+\varepsilon q)k_x^2} \tag{32}$$

where $q = 1 + 2\varepsilon$, and ε i δ are Tomsen's parameters [4]

$$\eta = 2(\varepsilon - \delta)$$

whereas $S_\perp$ is the slowness in the perpendicular direction to the layers and is expressed by relation $S_\perp = V_\perp^{-1}$, $V_\perp$ is the velocity of longitudinal wave in perpendicular direction to lamination. The knowledge of the vertical wavenumber k_z is an essential element of algorithm MG(F-K) of migration in dual frequency domain (F) and wavenumbers and space-time domain *x-t* [4, 5]. The two-stage process of wavefield extrapolation assumes its relocation according to relation:

$$U'(k_x, z_j + \Delta z, \omega) = e^{-ik_{z_o}\Delta_z} U(k_x, z_j, \omega) \tag{33}$$

from level z_j to level $z_j + \Delta z$ by means of exponential operator with vertical wavenumber k_{z_o} corresponding to homogeneous medium. At the second stage a correction is made by means of filter $F_j(x,\omega) = [1 - i/2\,\Delta z M_{j(x)}]^{-1}$ – sums of Neumann power series, where:

$$M_j(x) = \int k_{z_o}^{-1}\left(k_{z_o}^2 - k_z^2\right)e^{ik_x x}dk_x \tag{34}$$

The correction is represented in this way:

$$U_{z_j+\Delta z} = F_j(x,\omega)U'(x, z_j + \Delta z, \omega) \tag{35}$$

where $U'(x, z_j + \Delta z, \omega)$ is the transform of function $U'(k_x, z_j + \Delta z, \omega)$.

The last relation presents corrected wavefield at the level $z_j + \Delta z$, which is then subjected to another iterative step in wavefield extrapolation into the depth of the medium.

In case of migration, before stack the extrapolation algorithm will be the product of corrective functions referred to the sources and receivers, while for the option zero-offset in relation (32) slowness $S_\perp$ should be multiplied by 2. The detailed method of proceeding is analogical to the one discussed in the article by A. Kostecki [3].

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Literature

[1] Han Q., Wu R.S.: *A one-way dual propagator for scalar qP waves in VTI medium*. Geophysics, vol. D-9-17,2005.

[2] Kostecki A.: *Algorithm MG(F-K) of migration in model TTI anisotropy*. Nafta-Gaz nr 1, s. 5, 2010.

[3] Kostecki A.: *Algorithms of depth migration in anisotropic medium VTI*. Nafta-Gaz nr 11, s. 661-667, 2007 (in polish).

[4] Kostecki A., Półchłopek A.: Seismic *migration before stack in lateral heterogenity medium.* Works by Oil and Gas Institute no. 94, 1998 (in polish).

[5] Kostecki A., Półchłopek A.: *Stable depth extrapolation of seismic wavefields by Neumann series*. Geophysics, 63, 2063–2071, 1998.

[6] Kostecki A.: *Algorithm of migration MG(F-K) for anisotropic medium type HTI (Horizontal Transversely Isotropy).* Nafta-Gaz nr 1, s. 5–9, 2010 (in polish).

[7] Rüger A.: *Reflection coefficients and azimuthal AVO analysis in anisotropic media*. Society of exploration geophysicists, USA 2002.

[8] Thomsen L.: *Weak elastic anisotropy*. Geophysics, vol. 51, 1954–1966, 1986.

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